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ON LONG-TERM PERIODICITIES IN THE SUNSPOT RECORD

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16. ABSTRACT

Sunspot records have been systematically maintained, with the knowledge that an 11-year average period exists, since about 1850. Thus, the sunspot record of highest quality and considered to be the most reliable is that of cycle 8 through the present. On the basis of cycles 8 through 20, we have used various combinations of sine curves to approximate the observed $\bar{R}_{MAX}$ values (where $\bar{R}_{MAX}$ is the smoothed sunspot number at cycle maximum). We find that a three-component sinusoidal function, having an 11-cycle and a 2-cycle variation on a 90-cycle periodicity, yields computed $\bar{R}_{MAX}$ values which fit, reasonably well, observed $\bar{R}_{MAX}$ values for the modern sunspot cycles. Extrapolation of the empirical function forward in time allows us to project values of $\bar{R}_{MAX}$ for cycles 21 and 22. For cycle 21, the function projects a value of 157.3, very close to the actually observed value of 164.5 and to that predicted earlier by Sargent [64] - 154. For cycle 22, the function projects a value of about 107. Linear regressions identified in Wilson [44] have been applied to cycle 22, yielding the result that it will probably be a long-period cycle (cycle duration > 132 months). A major feature of this report is an extensive bibliography on techniques used to estimate the time-dependent behavior of sunspot cycles.

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LIST OF SELECTED ABBREVIATIONS

Abbreviation	Meaning
ASC	Ascent cycle duration in months
ASC_{CHM}	Time in months from $\overline{R}_{MIN}$ to $\overline{R}_{MAX}$; CHM means "corrected half maximum"
D_{CHM}	Time in months when $\overline{R}_{13} \geq \overline{R}_{CHM}$
DES	Descent cycle duration in months
LR	Linear Regression
MIN-MIN PERIOD	Minimum-to-minimum cycle duration; ASC + DES
N	Number
$\overline{R}_{13}$	Smoothed sunspot number; monthly moving average of sunspot number
R(A)	Annual mean sunspot number
$\overline{R}_{CHM}$	$(\overline{R}_{MAX} + \overline{R}_{MIN})/2$
$\overline{R}_{MAX}$	$\overline{R}_{13}$ value at cycle maximum
$\overline{R}_{MAX}(COMP)$	Computed $\overline{R}_{MAX}$
$\overline{R}_{MAX}(OBS)$	Observed $\overline{R}_{MAX}$
$\overline{R}_{MEAN}$	Mean $\overline{R}_{13}$ value for entire cycle
$\overline{R}_{MIN}$	$\overline{R}_{13}$ value at cycle minimum
s	Standard deviation
s_{MEAN}	Standard deviation associated with $\overline{R}_{MEAN}$
$SLOPE_{ASC}$	$(\overline{R}_{MAX} - \overline{R}_{MIN})/ASC$, where all parameters are for same cycle
$SLOPE_{DES}$	$(\overline{R}_{MIN} - \overline{R}_{MAX})/DES$, where $\overline{R}_{MAX}$ and DES are for a particular cycle and $\overline{R}_{MIN}$ is for the succeeding cycle

TECHNICAL MEMORANDUM

ON LONG-TERM PERIODICITIES IN THE SUNSPOT RECORD

I. INTRODUCTION

In recent years, substantial interest has been given to the sunspot record, especially its historical aspects. Much of this work sought to associate possible climatic excursions with variations in sunspot number. For example, Eddy [1-8] has done considerable research on the historical record of sunspot data and solar activity (also [9-43, 162-167, 172-183]). His studies, based primarily on naked-eye and early telescopic sunspot observations, fossil radiocarbon levels in tree rings, and auroral reports, have suggested possibly strong climatic associations in the Sun-Earth record; in particular, he has found evidence for the occurrence of prolonged "sunspot minima," e.g., the Spörer Minimum (A.D. 1400-1510) and Maunder Minimum (A.D. 1645-1715), and for the occurrence of a prolonged "sunspot maximum," i.e., the so-called Medieval Maximum or Grand Maximum (ca., A.D. 1120-1280). In addition, he has given tutorial remarks concerning the sunspot record post-1850 and pre-1850; i.e., he notes that the highest or best-quality sunspot data are those dated from about 1850 to the present, since it was about 1850 when solar observers first began to record telescopic sunspot observations in a systematic fashion, having just become aware of the existence of the cyclic nature of sunspots (Wilson [44] and Wilson et al. [45]). Sunspot observations prior to about 1850 are viewed as being somewhat less reliable, or at least somewhat different, as a simple comparison of selected cycle parameters for pre-1850 to those post-1850 will show. (Sonett [36] has suggested that there may be "bookkeeping" type errors in the compilations associated with these early sunspot cycles; also it must not be forgotten that daily sunspot records were not maintained on a continuous basis until January 1849, Waldmeier [170].) Cycles have been arbitrarily assigned numbers back in time into the 1700's (and before), with sunspot cycle number (SCN) 1 having a maximum in June 1761; the present cycle, now in decline, is denoted SCN 21.

Another aspect of this work has been to estimate or "predict" sunspot numbers as a function of time for future epochs, based on modeling or "post-dicting" of past cycles. This aspect usually incorporates various time series techniques which sometimes make use of power-spectrum analysis results. Power-spectrum analyses suggest the existence of several possible short-term and long-term periodicities in the solar cycles; e.g., several authors have reported the presence of 5-, 8-, 10-, 22-, 90-, and 180-year periods, as well as the well-known 11-year average period cycle duration in the sunspot record. Some authors have hypothesized that these periods may be due to planetary motion "tidal" effects, while others argue that sunspot-like cycles can be simulated with random noise. It must be pointed out, however, that some skepticism remains regarding the authenticity of most of these short-term and long-term periodicities, and, following Wallenhorst [62], it is important to note that the 11-year periodicity is the only statistically significant one present in the sunspot data. We should recall that sunspots have been routinely and systematically monitored for a mere 130 years or so; thus, few or no cycles of periods substantially longer than 11 years have been observed. Wilson [44], Wallenhorst [46], Newkirk and Frazier [47], Bray and Loughhead [48], Noyes [49], and Meadows [50] have given additional comments regarding sunspot numbers, their history, measurement, and use.

A number of references [51-161, 170-183] are included which allows the reader to follow the past 80 years growth in our understanding of sunspot cyclicity and to see the methods employed for estimating contemporary values of sunspot number.

The purpose of this report is threefold: (1) to compare observed sunspot numbers at cycle maximum of contemporary cycles (i.e., cycles 8 through 20) with similar computed values, based on the assumption that sunspots are periodic in nature (i.e., sinusoidal); (2) to extrapolate the preferred, empirical periodic function "backwards" in time to qualitatively compare computed values with values estimated by others for such epochs as the Maunder Minimum; and (3) to extrapolate the preferred periodic function "forwards" in time to estimate smoothed sunspot number at sunspot maximum (denoted, $\bar{R}_{MAX}$) for cycles 21, 22, and beyond. In addition, using linear relationships identified in Wilson [44], a number of cycle-related parametric values will be estimated and compared with observed values (when they are known) for cycle 21 and will be projected for cycle 22. In Section II, the approach used in this study is described; in Section III, the results and discussion of the findings are presented; and in Section IV, the conclusions are stated. This report is a continuation of studies investigating solar activity and its estimate (e.g., Wilson [44, 102], Wilson et al. [45, 100], and Teuber et al. [107]).

II. APPROACH

Since Eddy [4] (and before him McNish and Lincoln [123]) has cautioned that cycles observed prior to about 1850, corresponding to SCN 9, may not be reliably known, many solar statisticians today limit their remarks concerning cycle-related parameters, such as ascent period (dubbed ASC, the time in months measured from $\bar{R}_{MIN}$ occurrence to $\bar{R}_{MAX}$ occurrence; $\bar{R}_{MIN}$ is the minimum smoothed sunspot number or $\bar{R}_{13}$ value marking the beginning of a cycle and $\bar{R}_{MAX}$ is the maximum $\bar{R}_{13}$ value in the same cycle), descent period (dubbed DES, the time in months from $\bar{R}_{MAX}$ occurrence to subsequent cycle $\bar{R}_{MIN}$ occurrence), cycle duration (denoted MIN-MIN PERIOD or the cycle minimum-to-minimum period, numerically equal to ASC + DES), and others to cycles 8 through 20. Table 1 lists values for the cycle-related parameters for cycles 8 through 20. It also gives mean and "uncorrected" standard deviation for each parameter. Definitions of these parameters are given in the list of selected abbreviations found at the beginning of this report.

Using $\bar{R}_{MAX}$ values for cycles 8 through 20, this report compares a linear-regression approach and modifications to it (APPROACH 1) with two cyclic approaches and modifications to them (APPROACH 2 and APPROACH 3). The two cyclic approaches presume a 50-cycle or approximately 550-year periodicity and a 90-cycle or 990-year periodicity in sunspot number, respectively. The reasoning behind the selection of these two periods will become evident in the following.

If restricted to cycles 8 through 20, it is observed that the mean value for $\bar{R}_{MAX}$ is 116.2 with a 1-sigma confidence value (or standard deviation, s) of 36.7. (For small sample size, N, when N < 50, the "corrected" 1-sigma value is equal to $\sqrt{N/(N-1)}$ x s, or 38.2.) Thus, any given cycle can "crudely" be predicted to have, assuming a 1-sigma confidence with no apparent trend in the data, an $\bar{R}_{MAX}$

TABLE 1. SELECTED VALUES FOR CYCLES 8 THROUGH 20

SCN	$\bar{R}_{MAX}$	$\bar{R}_{MIN}$	$\bar{R}_{MEAN}$	s_{MEAN}	$\bar{R}_{CHM}$	ASC_{CHM}	D_{CHM}	ASC	DES	MIN–MIN PERIOD	$SLOPE_{ASC}$	$SLOPE_{DES}$
8	146.9	7.3	66.9	43.4	77.1	23	53	40	76	116	3.490	−1.795
9	132.0	10.5	55.9	30.4	71.3	44	39	55	94	149	2.209	−1.370
10	97.9	3.2	48.2	29.6	50.6	29	57	50	85	135	1.894	−1.091
11	140.5	5.2	52.9	43.8	72.9	28	46	41	100	141	3.300	−1.383
12	74.6	2.2	33.8	23.8	38.4	22	63	60	74	134	1.207	−0.941
13	87.9	5.0	38.5	28.1	46.5	20	54	47	96	143	1.764	−0.888
14	64.2	2.7	32.1	22.3	33.5	23	72	49	89	138	1.255	−0.704
15	105.4	1.5	44.0	30.5	53.5	27	50	49	71	120	2.120	−1.406
16	78.1	5.6	40.3	25.6	41.9	24	59	57	65	122	1.272	−1.148
17	119.2	3.5	57.2	36.9	61.4	29	57	43	82	125	2.691	−1.360
18	151.8	7.7	74.0	50.5	79.8	27	51	39	83	122	3.695	−1.788
19	201.3	3.4	90.5	67.6	102.4	23	54	47	79	126	4.211	−2.427
20	110.6	9.6	59.9	34.8	60.1	23	74	49	91	140	2.061	−1.081
$MEAN_{8-20}$	116.2	5.2	53.4	35.9	60.7	26.3	56.1	48.2	83.5	131.6	2.398	−1.337
s_{8-20}	36.7	2.7	16.2	–	18.8	5.8	9.2	6.2	10.0	10.0	0.958	0.441

value equal to approximately 116.2 ± 36.7. Statistically, correct prediction would be expected about 68 percent of the time. A 2–sigma confidence, or $\bar{R}_{MAX} = 116.2 \pm 73.4$, should result in being correct about 95 percent of the time. Therefore, it would be extremely useful to deduce a relationship which gives better estimates for $\bar{R}_{MAX}$ with reduced 1–sigma spreads. In APPROACH 1, $\bar{R}_{MAX}$ values are "detrended" for cycles 8 through 20 by means of standard linear–regression time series analysis (e.g., Downie and Heath [168] and Longley–Cook [169]).

Examination of $\bar{R}_{MAX}$ values for cycles 8 through 20 clearly shows a downward trend between cycles 8 through about 14 and an upward trend from cycle 14 through at least cycle 19. The entire period spanning cycles 8 through 20 shows an upward trend. Based purely on the deduced regression equation, $\bar{R}_{MAX}$ can be "better" predicted since s has been reduced to 33.2.

Various modifications will be made on this linear–regression fit (denoted LR) to further reduce s. These modifications include assuming the presence of an 8–cycle periodicity or an 11–cycle periodicity, an 11–cycle periodicity with an imbedded 8–cycle periodicity, and an 11–cycle periodicity with an imbedded 2–cycle periodicity. The 8–cycle periodicity was selected because it is equivalent to the so-called "Gleissberg" 90-year period often said to be imbedded in the sunspot data [55]. The 11–cycle periodicity is the "best-fit" periodicity for the given $\bar{R}_{MAX}$ values (cycles 8 through 20). (The term "best-fit" is used extensively in this report. By this we mean that the fit to the observed values of $\bar{R}_{MAX}$ has the least sum of the squares of the residuals; see Section III.) The 2–cycle periodicity was selected since throughout much of the period spanning cycles 8 through 20, there is an apparent "up-down-up" signal in the data.

In APPROACH 2, $\bar{R}_{MAX}$ values are fitted for cycles 8 through 20 using a sine curve with a period equal to 50 cycles. This period was chosen based on the assumption that the Maunder Minimum actually occurred (the middle of the minimum corresponds to SCN -6) and that the sine curve is at phase 270 deg at that point and at phase 90 deg at SCN 19; hence, the half-period of the sine curve is 25-cycles, yielding the period to be 50 cycles. Such an analysis gives a 1-sigma value of 37.0 and yields a line that is very close in value to the linear-regression line. Modifications to this approach are as above in APPROACH 1, using various combinations of an 11-cycle, 8-cycle, and/or 2-cycle periodicity imbedded in the 50-cycle fit.

In APPROACH 3, the LR fit is approximated by a sine curve of period equal to 90 cycles, such that the curve is at phase 90 deg during the Grand Maximum (at SCN -53) and at phase 270 deg during the Maunder Minimum (at SCN -8). Such an analysis gives a 1-sigma value of 36.1 and yields a line very close to that of the regression line. Modifications to this approach follow that above for APPROACH 1 and APPROACH 2, using various combinations of an 11-cycle, 8-cycle, and/or 2-cycle periodicity imbedded in the 90-cycle fit.

III. RESULTS AND DISCUSSION

Following the approaches identified in the previous section, several fits to the observed $\bar{R}_{MAX}$ values for cycles 8 through 20 are obtained. These basic, first-order fits are shown in Figure 1. The observed values are plotted as darkened circles and the computed values as darkened triangles. The lines shown are: mean fit, linear-regression fit, 50-cycle fit, and 90-cycle fit. Equations are given below and in Figure 1 for these fits:

Mean Fit

$$\bar{R}_{MAX} = 116.2 \tag{1}$$

Linear-Regression Fit

$$\bar{R}_{MAX} = 88.1 + 2.0\ X \tag{2}$$

50-Cycle Fit

$$\bar{R}_{MAX} = 100.7 + 18.7\ \sin\theta_1(X) \tag{3}$$

90-Cycle Fit

$$\bar{R}_{MAX} = 116.0 + 24.7\ \sin\theta_5(X) \tag{4}$$

In equation (2), X is the sunspot cycle number (SCN). In equations (3) and (4), $\theta_1(X)$ is the angle in degrees for a particular X assuming a 50-cycle periodicity, and

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$\bar{R}_{MAX}$ VALUES VS. SCN

MEAN FIT: $\bar{R}_{MAX} = 116.2$, $S = 36.7$
LINEAR REGRESSION FIT:
$\bar{R}_{MAX} = a_0 + a_1 X$, $a_0 = 88.1$, $a_1 = 2.0$
$X = SCN$, $S = 36.1$
50−CYCLE FIT: $\bar{R}_{MAX} = a_0 + a_1 \sin \theta_1 (X)$
$a_0 = 100.7$, $a_1 = 18.7$, $S = 37.0$
$\theta_1 (X)$: SEE TABLE
● OBSERVED VALUE
▲ COMPUTED VALUE

LINEAR−REGRESSION FIT
50−CYCLE FIT
MEAN FIT

$\bar{R}_{MAX}$ VALUE

SUNSPOT CYCLE NUMBER (SCN)

$\bar{R}_{MAX}$ VALUES VS SCN

90−CYCLE FIT = $\bar{R}_{MAX} = a_0 + a_1 \sin \theta_5 (X)$
$a_0 = 116.0$, $a_1 = 24.7$, $S = 36.1$
$\theta_5 (X)$ = SEE TABLE
● OBSERVED VALUE
▲ COMPUTED VALUE

LR − FIT
90−CYCLE FIT

$\bar{R}_{MAX}$ VALUE

SUNSPOT CYCLE NUMBER (SCN)
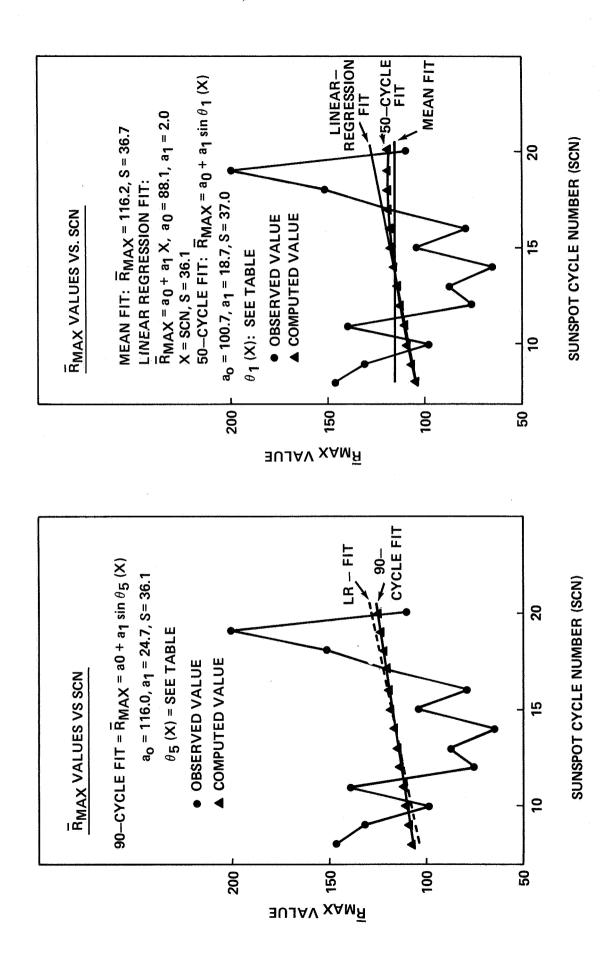
Figure 1. $\bar{R}_{MAX}$ values versus SCN (cycles 8 through 20).

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$\theta_5(X)$ is the angle in degrees for a particular X assuming a 90-cycle periodicity, respectively. These angular values are identified in Table 2. The standard deviations for each of these fits are 36.7, 36.1, 37.0, and 36.1, respectively. Equation (3) is of the form $y = a_0 + a_1 \sin\theta$, where $a_0 = 100.7$ and $a_1 = 18.7$. The constant a_0 was chosen to be 100.7 on the basis that SCN 19 has the highest known $\bar{R}_{MAX}$ value (= 201.3) and that the lowest possible value for any cycle is 0; thus, a_0 is the rounded-off half-way point between these two extremes. The constant a_1 is the "best-fit" value based on the given a_0 and the values of $\theta_1(X)$ as shown in Table 2 for cycles 8 through 20. The "best-fit" a_1 was deduced by arbitrarily selecting three values of a_1 and computing $\bar{R}_{MAX}$ values for cycles 8 through 20 with the given a_0, summing the squares of the residuals (where a residual is the difference between the observed and computed value of $\bar{R}_{MAX}$ for a cycle), and fitting these sums of squares of the residuals versus a_1 which yields the least value for the sums of the squares of the residuals.

TABLE 2. ANGULAR VALUES (PHASES) FOR SELECTED PERIODIC
CYCLES AS A FUNCTION OF SCN

SCN	CYCLE MAXIMUM APPROXIMATE YEAR DATE	50-CYCLE $\theta_1 X$	11-CYCLE $\theta_2 X$	8-CYCLE $\theta_3 X$	2-CYCLE $\theta_4 X$	90-CYCLE $\theta_5 X$	NOTES
−60	1088	241.2	24.5	135	270	62	
−59	1099	248.4	57.3	180	90	66	
−58	1110	255.6	90.0	225	270	70	
−57	1121	262.8	122.7	270	90	74	(APPROXIMATE)
−56	1132	270.0	155.5	315	270	78	
−55	1143	277.2	188.2	360	90	82	
−54	1154	284.4	220.9	45	270	86	
−53	1165	291.6	253.6	90	90	90	
−52	1176	298.8	286.4	135	270	94	GRAND MAXIMUM
−51	1187	306.0	319.1	180	90	98	
−50	1198	313.2	351.8	225	270	102	
−49	1209	320.4	24.5	270	90	106	
−48	1220	327.6	57.3	315	270	110	
−47	1231	334.8	90.0	360	90	114	
−46	1242	342.0	122.7	45	270	118	
−45	1253	349.2	155.5	90	90	122	
−44	1264	356.4	188.2	135	270	126	
−43	1275	3.6	220.9	180	90	130	
−42	1286	10.8	253.6	225	270	134	(APPROXIMATE)
−41	1297	18.0	286.4	270	90	138	
−40	1308	25.2	319.1	315	270	142	
−39	1319	32.4	351.8	360	90	146	
−38	1330	39.6	24.5	45	270	150	
−37	1341	46.8	57.3	90	90	154	
−36	1352	54.0	90.0	135	270	158	
−35	1363	61.2	122.7	180	90	162	
−34	1374	68.4	155.5	225	270	166	
−33	1385	75.6	188.2	270	90	170	
−32	1396	82.8	220.9	315	270	174	
−31	1407	90.0	253.6	360	90	178	
−30	1418	97.2	286.4	45	270	182	(APPROXIMATE)
−29	1429	104.4	319.1	90	90	186	
−28	1440	111.6	351.8	135	270	190	
−27	1451	118.8	24.5	180	90	194	
−26	1462	126.0	57.3	225	270	198	SPÖRER
−25	1473	133.2	90.0	270	90	202	MINIMUM
−24	1484	140.4	127.7	315	270	206	
−23	1495	147.6	155.5	360	90	210	
−22	1506	154.8	188.2	45	270	214	
−21	1517	162.0	220.9	90	90	218	(APPROXIMATE)
−20	1528	169.2	253.6	135	270	222	
−19	1539	176.4	286.4	180	90	226	
−18	1550	183.6	319.1	225	270	230	
−17	1561	190.8	351.8	270	90	234	
−16	1572	198.0	24.5	315	270	238	

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TABLE 2. (Concluded)

SCN	CYCLE MAXIMUM APPROXIMATE YEAR DATE	50-CYCLE $\theta_1 X$	11-CYCLE $\theta_2 X$	8-CYCLE $\theta_3 X$	2-CYCLE $\theta_4 X$	90-CYCLE $\theta_5 X$	NOTES
−15	1583	205.2	57.3	360	90	242	
−14	1594	212.4	90.0	45	270	246	
−13	1605	219.6	122.7	90	90	250	
−12	1616	226.8	155.5	135	270	254	
−11	1627	234.0	188.2	180	90	258	
−10	1638	241.2	220.9	225	270	262	(APPROXIMATE)
− 9	1649	248.4	253.6	270	90	266	
− 8	1660	255.6	286.4	315	270	270	MAUNDER MINIMUM
− 7	1671	262.8	319.1	360	90	274	
− 6	1682	270.0	351.8	45	270	278	
− 5	1693	277.2	24.5	90	90	282	
− 4	1704	284.4	57.3	135	270	286	(APPROXIMATE)
− 3	1715	291.6	90.0	180	90	290	
− 2	1726	298.8	122.7	225	270	294	
− 1	1737	306.0	155.5	270	90	298	
0	1748	313.2	188.2	315	270	302	
1	1759	320.4	220.9	360	90	306	
2	1770	327.6	253.6	45	270	310	
3	1781	334.8	286.4	90	90	314	
4	1792	342.0	319.1	135	270	318	
5	1803	349.2	351.8	180	90	322	
6	1814	356.4	24.5	225	270	326	
7	1825	3.6	57.3	270	90	330	BEST−QUALITY SUNSPOT DATA
8	1836	10.8	90.0	315	270	334	
9	1847	18.0	122.7	360	90	338	
10	1858	25.2	155.5	45	270	342	
11	1869	32.4	188.2	90	90	346	
12	1880	39.6	220.9	135	270	350	
13	1891	46.8	253.6	180	90	354	
14	1902	54.0	286.4	225	270	358	
15	1913	61.2	319.1	270	90	2	
16	1924	68.4	351.8	315	270	6	
17	1935	75.6	24.5	360	90	10	
18	1946	82.8	57.3	45	270	14	
19	1957	90.0	90.0	90	90	18	
20	1968	97.2	122.7	135	270	22	
21	1979	104.4	155.5	180	90	26	
22	1990	111.6	188.2	225	270	30	
23	2001	118.8	220.9	270	90	34	
24	2012	126.0	253.6	315	270	38	
25	2023	133.2	286.4	360	90	42	
26	2034	140.4	319.1	45	270	46	
27	2045	147.6	351.8	90	90	50	
28	2056	154.8	24.5	135	270	54	
29	2067	162.0	57.3	180	90	58	
30	2078	169.2	90.0	225	270	62	

Similarly, equation (4) is of the form $y = a_0 + a_1 \sin\theta$, where $a_0 = 116$ and $a_1 = 24.7$. The constant a_0 was selected to be approximately the value that the linear-regression equation [eq. (2)] yields for SCN 14, the mid-point of the data set. As above, a_1 is the "best-fit" for the given a_0 and the values of $\theta_5(X)$ shown in Table 2, using the least-squares method.

In Figure 2, the residuals are plotted in units of standard deviation for each of the fits shown in Figure 1. For convenience, the figure is arbitrarily divided into several smaller regions. The middle region of each plot is designated "average" since approximately 50 percent, on average, of the $\bar{R}_{MAX}$ values are expected to lie within its bounds; the region just above (and below) "average" is called "above (below)
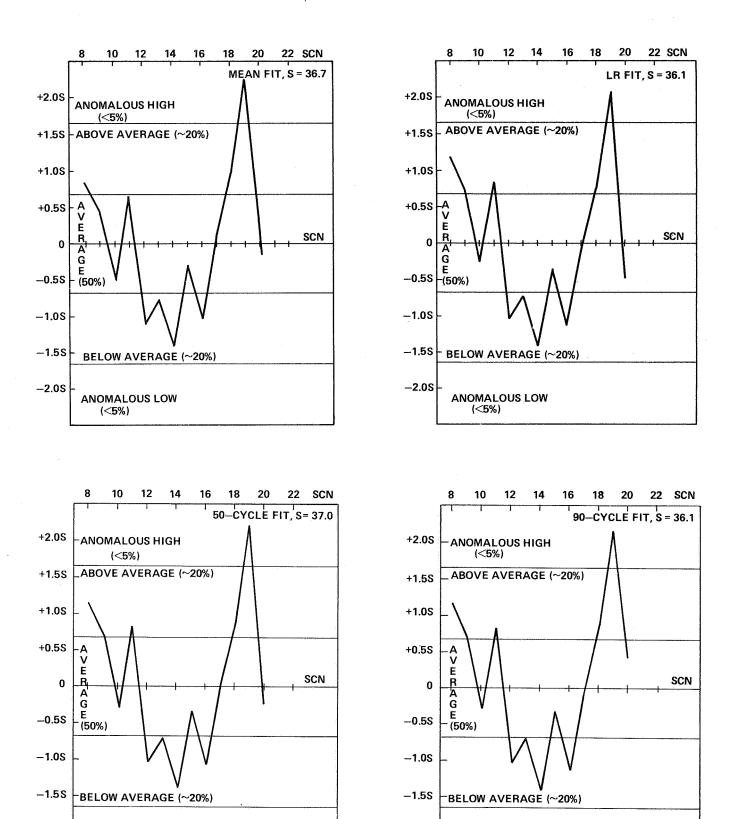
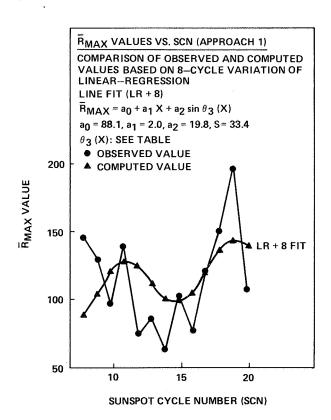Figure 2. Statistical spread versus SCN (cycles 8 through 20).

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average and approximately 20 percent, on average, of the cycles are expected to fall within its bounds. The top-most (bottom-most) region is called "anomalous high (low)" and represents the approximately upper (lower) 5 percent, on average, of the $\bar{R}_{MAX}$ values which are greater than 1.645 standard deviations above (below) the computed value for $\bar{R}_{MAX}$. Based on these single-component fits, SCN 19 is observed to probably be an anomalously high $\bar{R}_{MAX}$ cycle.

In subsections A, B, and C which follow, the effects of selected modifications to the aforementioned basic curve-fits (linear regression, 50-cycle, and 90-cycle fits) are examined. In subsection D, computed values of $\bar{R}_{MAX}$ are compared using the preferred empirical fit with values reported for earlier epochs (SCN 1 through 7 and the Maunder Minimum). (The reader is reminded that phase for the empirical fit is based, in part, on the occurrence of the Maunder Minimum; however, amplitude for the fit is based only on cycles 8 through 20.) In Subsection E, the empirical fit is used to infer values of $\bar{R}_{MAX}$ for future epochs (SCN 21, 22, and beyond).

A. Linear-Regression Fits (APPROACH 1)

Following the first modification in APPROACH 1, Figure 3 shows the linear-regression fit with an 8-cycle variation. The equation for this fit is:

$$\bar{R}_{MAX} = 88.1 + 2.0\ X + 19.8\ \sin\ \theta_3(X) \quad , \tag{5}$$
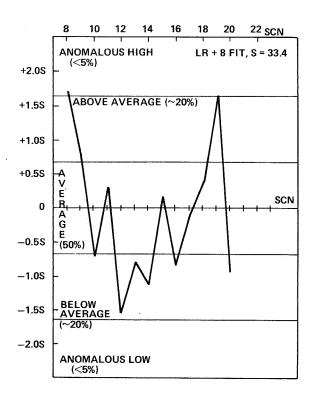

Figure 3. LR + 8 FIT.

where X is again SCN and $\theta_3(X)$ is found in Table 2. The constant 19.8 is the "best-fit" for the values given in the table and above. This fit only slightly reduces the statistical spread (s = 33.4 for the LR + 8 FIT as compared to s = 36.1 for the LR FIT alone); so, if indeed there is an 8-cycle periodicity in the $\bar{R}_{MAX}$ record for cycles 8 through 20, it does not appear to contribute much to the variation in $\bar{R}_{MAX}$ value with time. Based on this two-component fit, both cycles 8 and 19 are observed to probably be anomalously high.
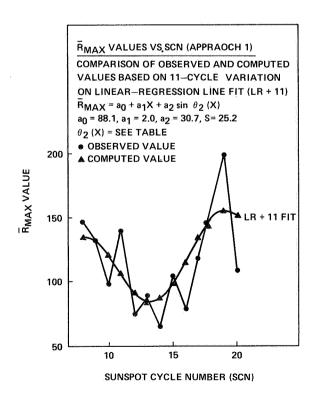
A better two-component fit is plotted in Figure 4 which shows the linear-regression fit with an 11-cycle variation. Its equation is:

$$\bar{R}_{MAX} = 88.1 + 2.0\ X + 30.7 \sin \theta_2(X) \quad . \tag{6}$$

As before, X is SCN and $\theta_2(X)$ can be found in Table 2. The constant 30.7 is the "best-fit" for the given table values and constants used in equation (6). This fit reduces the statistical spread by about 30 percent (s = 25.2 for the LR + 11 FIT). Based on this fit, cycle 19 is anomalously high and cycle 20 is anomalously low.

Figure 5 shows a three-component fit based on the linear-regression fit with an 11-cycle and 8-cycle variation. Its equation is:

$$\bar{R}_{MAX} = 88.1 + 2.0\ X + 30.7 \sin \theta_2(X) + 11.4 \sin \theta_3(X) \quad . \tag{7}$$
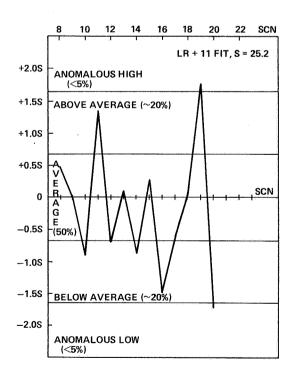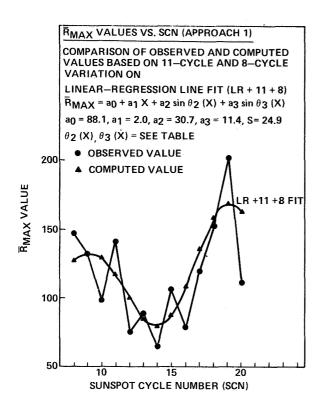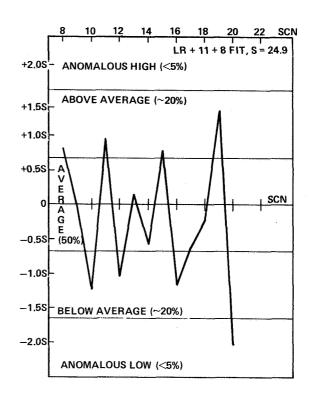

Figure 4. LR + 11 FIT.

Figure 5. LR + 11 + 8 FIT.

Again, this equation only slightly reduces the statistical spread as compared to equation (6) (s = 24.9 for the LR + 11 + 8 FIT). Based on this fit, cycle 20 is anomalously low.

The final fit following APPROACH 1 is shown in Figure 6. It is a three-component fit based on the linear-regression fit with an 11-cycle and 2-cycle variation. Its equation is:
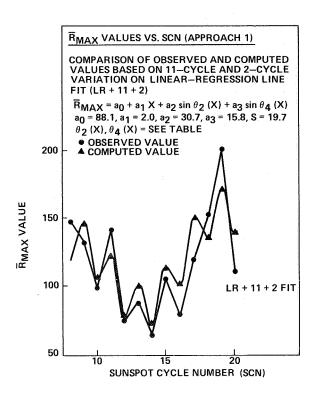
$$\bar{R}_{MAX} = 88.1 + 2.0 \ X + 30.7 \ \sin \ \theta_2(X) + 15.8 \ \sin \ \theta_4(X) \quad . \tag{8}$$

This fit reduces the original statistical spread from s = 36.1 to s = 19.7, a 45-percent reduction. (The variance has been reduced by 70 percent.) No cycles are observed to be anomalously low or high based on this fit, although several are in the above average or below average categories.

B. 50-Cycle Fits (APPROACH 2)

In APPROACH 2, the baseline fit is a 50-cycle periodicity. Modifications to it will be made as in APPROACH 1. Figure 7 depicts an 8-cycle variation on the 50-cycle fit. Its equation is:

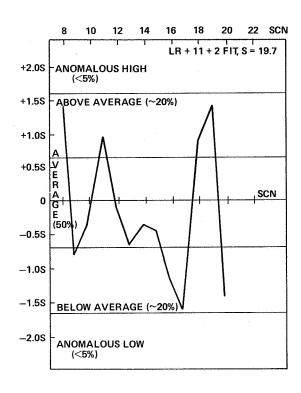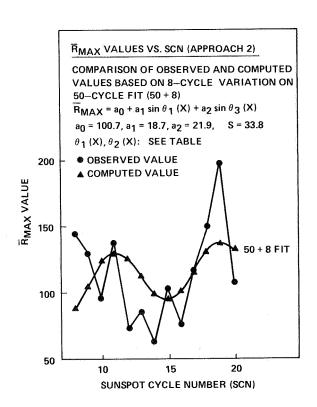$$\bar{R}_{MAX} = 100.7 + 18.7 \ \sin \ \theta_1(X) + 21 \ \sin \ \theta_3(X) \quad . \tag{9}$$

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Figure 6 (left panel):

$\bar{R}_{MAX}$ VALUES VS. SCN (APPROACH 1)

COMPARISON OF OBSERVED AND COMPUTED VALUES BASED ON 11–CYCLE AND 2–CYCLE VARIATION ON LINEAR–REGRESSION LINE FIT (LR + 11 + 2)

$\bar{R}_{MAX} = a_0 + a_1 X + a_2 \sin \theta_2 (X) + a_3 \sin \theta_4 (X)$
$a_0 = 88.1, a_1 = 2.0, a_2 = 30.7, a_3 = 15.8, S = 19.7$
$\theta_2 (X), \theta_4 (X) = $ SEE TABLE

● OBSERVED VALUE
▲ COMPUTED VALUE

$\bar{R}_{MAX}$ VALUE

LR + 11 + 2 FIT

SUNSPOT CYCLE NUMBER (SCN)

Figure 6 (right panel):

SCN

LR + 11 + 2 FIT, S = 19.7

+2.0S — ANOMALOUS HIGH (<5%)
+1.5S — ABOVE AVERAGE (~20%)
+1.0S
+0.5S
AVERAGE (50%)
0 — SCN
−0.5S
−1.0S
−1.5S — BELOW AVERAGE (~20%)
−2.0S — ANOMALOUS LOW (<5%)

Figure 6. LR + 11 + 2 FIT.

Figure 7 (left panel):

$\bar{R}_{MAX}$ VALUES VS. SCN (APPROACH 2)

COMPARISON OF OBSERVED AND COMPUTED VALUES BASED ON 8–CYCLE VARIATION ON 50–CYCLE FIT (50 + 8)

$\bar{R}_{MAX} = a_0 + a_1 \sin \theta_1 (X) + a_2 \sin \theta_3 (X)$
$a_0 = 100.7, a_1 = 18.7, a_2 = 21.9, \quad S = 33.8$
$\theta_1 (X), \theta_2 (X):$ SEE TABLE

● OBSERVED VALUE
▲ COMPUTED VALUE

$\bar{R}_{MAX}$ VALUE

50 + 8 FIT

SUNSPOT CYCLE NUMBER (SCN)

Figure 7 (right panel):

SCN

50 + 8 FIT, S = 33.8

+2.0S — ANOMALOUS HIGH (<5%)
+1.5S — ABOVE AVERAGE (~20%)
+1.0S
+0.5S
AVERAGE (50%)
0 — SCN
−0.5S
−1.0S
−1.5S — BELOW AVERAGE (~20%)
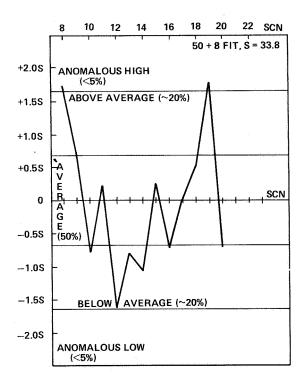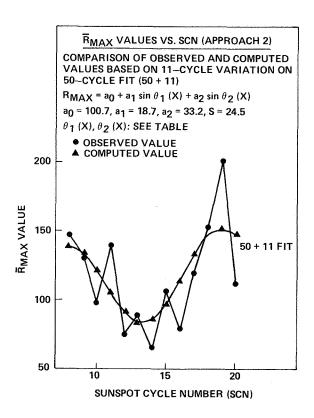−2.0S — ANOMALOUS LOW (<5%)

Figure 7. 50 + 8 FIT.

The statistical spread about this line is only slightly better than the 50-cycle fit alone (s = 33.8 for the 50 + 8 FIT). Cycles 8 and 19 both appear as anomalously high cycles.

In Figure 8, an 11-cycle variation on the 50-cycle fit is plotted. As in APPROACH 1, a much better fit is suggested. Its equation is:

$$\overline{R}_{MAX} = 100.7 + 18.7 \sin \theta_1(X) + 33.2 \sin \theta_2(X) \quad . \tag{10}$$

Its statistical spread is s = 24.5, about 34 percent more restrictive than the 50-cycle fit alone. Cycle 19 is the only cycle that appears to be anomalously high.
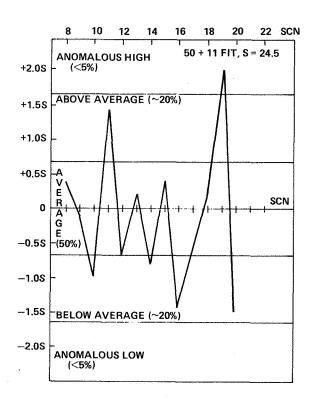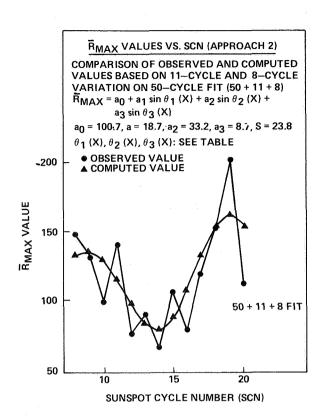
Figure 8. 50 + 11 FIT.

In Figure 9, an 11-cycle and an 8-cycle variation have been placed on the 50-cycle fit. Its equation is:

$$\overline{R}_{MAX} = 100.7 + 18.7 \sin \theta_1(X) + 33.2 \sin \theta_2(X) + 8.7 \sin \theta_3(X) \quad . \tag{11}$$

As before, the presence of an 8-cycle periodicity has little effect upon the reduction of the statistical spread (s = 23.8 for the 50 + 11 + 8 FIT). Cycles 19 and 20 appear to be anomalously high and low, respectively.
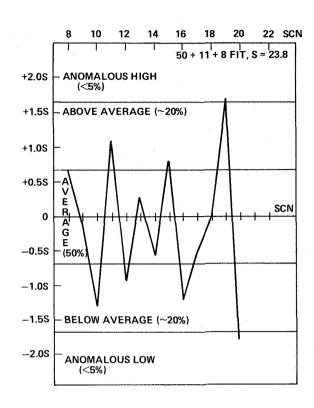
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Figure 9. 50 + 11 + 8 FIT.

In Figure 10, an 8-cycle and 2-cycle variation has been included with the 50-cycle periodicity. Its equation is:

$$\bar{R}_{MAX} = 100.7 + 18.7 \sin \theta_1(X) + 21.9 \sin \theta_3(X) + 12.8 \sin \theta_4(X) \quad . \tag{12}$$

This fit is worse than the preceding fit, having s = 31.2. Cycle 8 appears to be an anomalously high cycle.
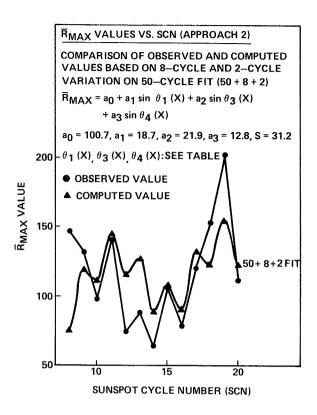
A much better fit is that shown in Figure 11 where an 11-cycle and a 2-cycle variation has been added to the baseline 50-cycle fit. Its equation is:

$$\bar{R}_{MAX} = 100.7 + 18.7 \sin \theta_1(X) + 33.2 \sin \theta_2(X) + 15.4 \sin \theta_4(X) \quad . \tag{13}$$

Its statistical spread is 49 percent smaller than that deduced for the baseline (s = 19.0 for the 50 + 11 + 2 FIT; the variance has been reduced by 74 percent). Only cycle 19 appears to be anomalously high.

C. 90-Cycle Fits (APPROACH 3)

In this subsection modifications to a 90-cycle periodicity, which approximates the linear-regression equation used in APPROACH 1, are examined. Figure 12 shows an 8-cycle variation on the 90-cycle fit. Its equation is:
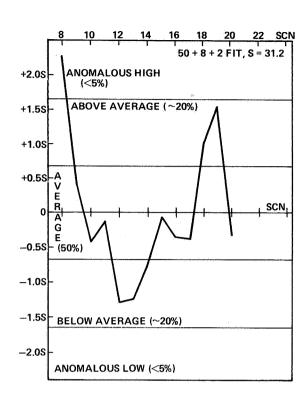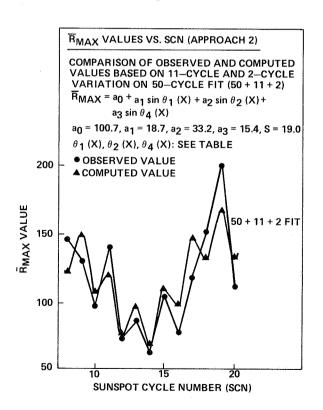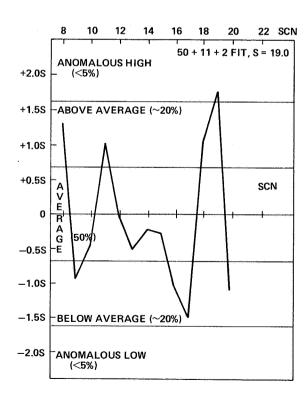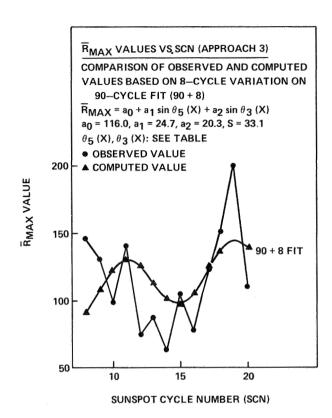
14

Figure 10.   50 + 8 + 2 FIT.
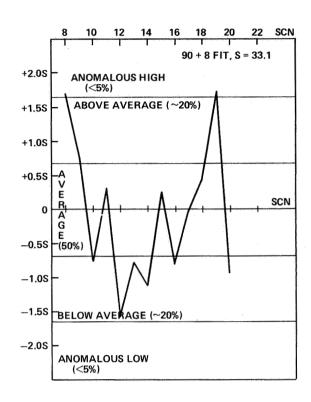

Figure 11.   50 + 11 + 2 FIT.

Figure 12.   90 + 8 FIT.

$$\overline{R}_{MAX} = 116.0 + 24.7 \sin \theta_5(X) + 20.3 \sin \theta_3(X) \quad . \tag{14}$$

Its spread is 33.1, only slightly reduced from the 90-cycle baseline spread of 36.1. Cycles 8 and 19 appear to be anomalously high.

A better fit is obtained using an 11-cycle variation on the 90-cycle periodicity, shown in Figure 13.   Its equation is:
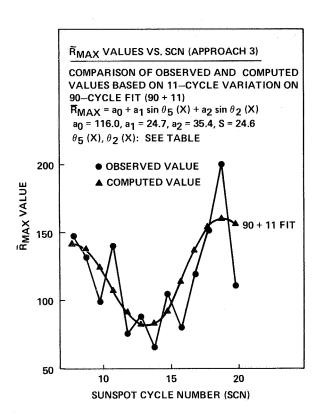
$$\overline{R}_{MAX} = 116.0 + 24.7 \sin \theta_5(X) + 35.4 \sin \theta_2(X) \quad . \tag{15}$$

Its 1-sigma spread is 24.6 units.   Cycles 19 and 20 appear to be anomalously high and low, respectively.

Figure 14 shows an 8-cycle and 2-cycle variation on the 90-cycle periodicity. Its equation is

$$\overline{R}_{MAX} = 116.0 + 24.7 \sin \theta_5(X) + 20.3 \sin \theta_3(X) + 13.2 \sin \theta_4(X) \quad . \tag{16}$$

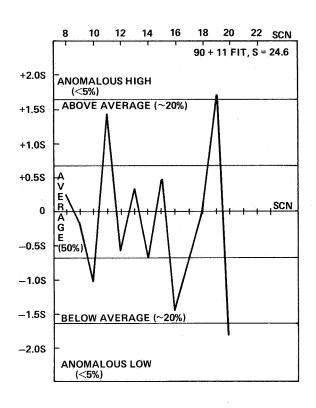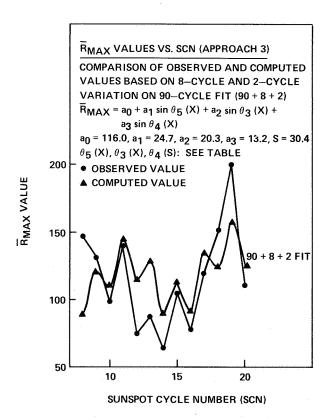Its 1-sigma spread is 30.4 and cycle 8 appears to be anomalously high.
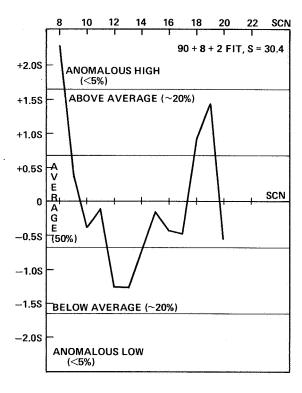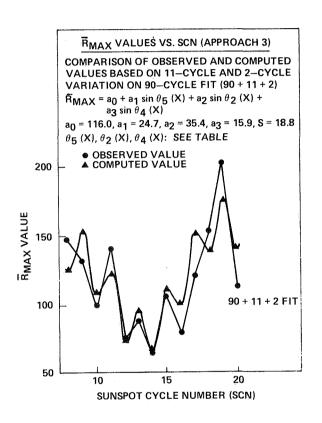
Figure 13. 90 + 11 FIT.

Figure 14. 90 + 8 + 2 FIT.

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Figure 15 depicts the function which had the smallest spread for the examples tested. It is an 11-cycle and 2-cycle variation on the 90-cycle periodicity. Its equation is:

$$\bar{R}_{MAX} = 116.0 + 24.7 \sin \theta_5(X) + 35.4 \sin \theta_2(X) + 15.9 \sin \theta_4(X) \quad . \quad (17)$$

Its 1-sigma spread is 18.8 units, a 48-percent reduction from the baseline spread. (The variance has been reduced by 73 percent.) Cycle 17 appears to be anomalously low.
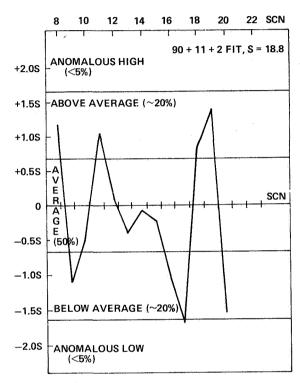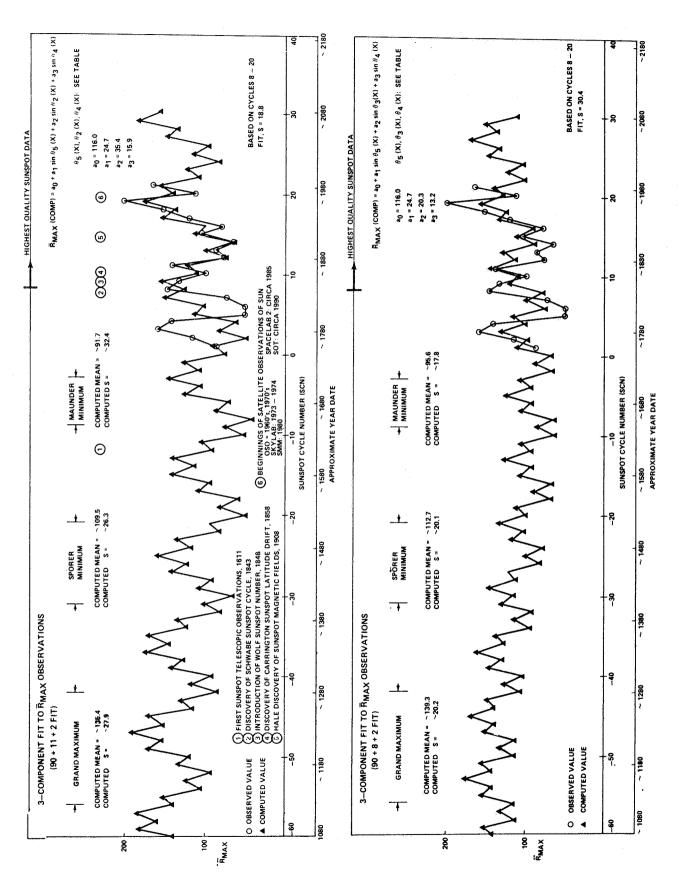
Figure 15. 90 + 11 + 2 FIT.

D. Pre-Cycle 8 Fits

According to the above analysis, values of $\bar{R}_{MAX}$ for cycles 8 through 20, the most reliably known, can be fit very well by equation (17). Using this preferred empirical function, one can now extrapolate "backwards" in time to determine computed smoothed sunspot number at cycle maximum for earlier epochs. (It is realized that equation (17) is merely a fit for a small sample, and that this fit may have no "post-dictive." or "pre-dictive" value.) In Figure 16, $\bar{R}_{MAX}$ is computed backwards in time to about A.D. 1080, corresponding to SCN -60. The top-most line is the 90 + 11 + 2 FIT and the bottom-most line is the 90 + 8 + 2 FIT, drawn for comparison. Observed values of $\bar{R}_{MAX}$ for cycle 1 through 21 are drawn as open circles; computed values

Figure 16. Summary chart: computed R_{MAX} values for SCN -64 to SCN 30.

are drawn as darkened triangles. The approximate dates, as stated by Eddy [3,4], for the Maunder Minimum, Spörer Minimum, and Grand Maximum are shown. A table legend is included which identifies some major milestones in solar research, as related to the study of sunspots. Computed mean values and standard deviations are shown for the two minima and single maximum periods. A note reminding us that the best-quality sunspot data extends from about cycle 8 to the present is also included. One quickly observes that, while the 90 + 11 + 2 FIT gives a very good representation of cycles 8 through 21, it gives a very poor match to cycles 2 through 7, sometimes departing by more than 3 sigma from computed values. Cycles 2 through 7 appear to be better-fitted by the 90 + 8 + 2 FIT. This seems to be supportive of the belief that there is an 8-cycle periodicity imbedded in the sunspot record; however, how much weight should be given to this view and even to the firmness of the $\bar{R}_{MAX}$ values for cycles 2 through 7 remain unclear. Additional investigation should be pursued.

Table 3 gives values for several selected cycle-related parameters for cycles 1 through 7. Mean values and standard deviations are given for the group cycles 1 through 7 and for the groups cycles 8 through 20 and 1 through 20. Clearly, when parametric values are compared for the different groupings, it is found that in some cases they are quite dissimilar; e.g., ASC and DES. This may be an indication that smoothed sunspot number at cycle maximum for cycles observed prior to cycle 8 are in error. (It is known from Waldmeier [170] that daily records of sunspot number, which form the basis for a determination of monthly mean sunspot number which is then used to compute smoothed sunspot number, are incomplete.) Rather, it may mean that the period encompassing cycles 2 through 7 was an anomalous timespan, with values departing from computed values by substantial amounts. (Very recently, Sonett [36] has addressed this particularly anomalous era and found the sunspot record to be consistent with the idea that, perhaps, there may have been a "book-keeping" error in the compilation of sunspot number about 1780 to 1800.)

TABLE 3. SELECTED PARAMETRIC VALUES FOR CYCLES 1 THROUGH 7

SCN	MINIMUM OCCURRENCE DATE	MAXIMUM OCCURRENCE DATE	$\bar{R}_{MIN}$	$\bar{R}_{MAX}$	ASC	DES	MIN—MIN PERIOD
1	FEB 1755	JUN 1761	8.4	86.5	75	60	135
2	MAY 1766	SEP 1769	11.2	115.8	39	69	108
3	JUN 1775	MAY 1778	7.2	158.5	35	76	111
4	SEP 1784	FEB 1788	9.5	141.2	41	96	137
5	APR 1798	FEB 1805	3.2	49.2	81	65	146
6	JUL 1810	MAY 1816	0.0	48.7	70	83	153
7	APR 1823	NOV 1829	0.1	71.7	79	48	127
$MEAN_{1-7}$			5.7	95.9	60.0	71.0	131.0
s_{1-7}			4.2	40.4	19.1	14.6	15.6
$MEAN_{8-20}$			5.2	116.2	48.2	83.5	131.6
s_{8-20}			2.7	36.7	6.2	10.0	10.0
$MEAN_{1-20}$			5.4	109.1	52.3	79.1	131.4
s_{1-20}			3.3	39.2	13.6	13.2	12.2

Of the three aforementioned, preceding epochs, only the Maunder Minimum can be examined more closely. According to equation (17), as Figure 16 shows, a minimum $\bar{R}_{MAX}$ value occurs in the 17th century. However, the values postdicted by equation (17) are higher than has been suggested for this period (e.g., Eddy [1,8]). (Equation (17) suggests a mean value for $\bar{R}_{MAX}$ of about 91 ± 32.) Recently, Xu and Jiang [35] re-assessed the 17th Century Chinese sunspot records and concluded that the inference to a Maunder Minimum may be incorrect (Link [23] and Schove [24]). They found sightings which allowed them to argue that solar cycles were continuing during the Maunder Minimum and that the average annual sunspot number at cycle maximum during this period was around 50 or less. Very recently, Zhang [37], based on Korean auroral records for the period A.D. 1507-1747, reports that solar activity during the Maunder Minimum may not have been as low as Eddy has claimed. Also, Hameed and Wyant [38] have found an approximate 23-year cycle in surface temperatures in central England during and after the Maunder Minimum, suggesting that if variation in climate may be partly due to solar activity, then solar activity may still have been very much apparent even during the Maunder Minimum. Figure 16 gives $\bar{R}_{MAX}$ values which can be converted to annual numbers for direct comparison. Figure 17 plots annual sunspot number at cycle maximum, R(A), versus $\bar{R}_{MAX}$,
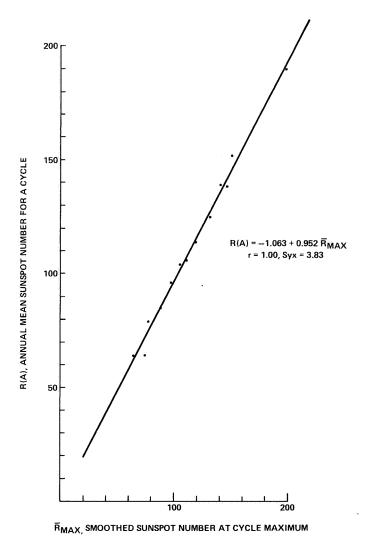

Figure 17. R(A) versus R_{MAX}.

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based on cycles 8 through 20. The Pearson correlation coefficient r is 1.00 with a standard error of estimate S_{yx} equal to about 3.8. The equation relating the two parameters is:

$$R(A) = -1.063 + 0.952 \, \overline{R}_{MAX} \quad . \tag{18}$$

Based on equation (18) and the 90 + 11 + 2 FIT, R(A) values are expected to have been about 86 ± 31 during the Maunder Minimum. If the Maunder Minimum is viewed as an anomalous timespan where R(A) values are depressed about 1 or more standard deviations below normal, then R(A) is expected to often be below 55, very near values reported by Xu and Jiang.

In Table 4, the Xu and Jiang results are recast along with those reported by Schove [22] and R(A) estimates based on the 90 + 11 + 2 and 50 + 11 + 2 FITS. The table spans SCN -14 to SCN -3, where the Maunder Minimum approximately spans SCN -9 to SCN -3. Six of the ten records (Xu and Jiang only give results for SCN -14 to SCN -5) for which Xu and Jiang give results are very compatible with those obtained using either the 90 + 11 + 2 or 50 + 11 + 2 FITS. If the statistical spread is allowed to be greater than 1-sigma, then, essentially all the records overlap. This comparison suggests that while a Maunder Minimum very well may have occurred, solar periodicities were probably still working and values of $\overline{R}_{MAX}$ given by the empirical function (depressed by at least 1-sigma, possibly more) may be representative of the period. (Recently, Eddy [8] has re-appraised the Maunder Minimum and argues that the basis for the occurrence of such a minimum remains firm, with peak annual sunspot number being of the order of 20 or below during this protracted period of low sunspot number.)

TABLE 4. COMPARISON OF REPORTED AND COMPUTED $\overline{R}_{MAX}$ VALUES FOR THE MAUNDER MINIMUM

SCN	R(A)$^1_{est}$	R(A)$^2_{est}$	R(A)$^3_{COMP}$	R(A)$^4_{COMP}$	NOTES
−14	50	80	106	102	
−13	>160	90	129	125	
−12	85	100	85	80	
−11	160	70	97	91	
−10	50	40	48	44	
− 9	50	50	68	63	
− 8	45		38	33	
− 7	< 45	60	79	71	
− 6	45	50	66	58	MAUNDER MINIMUM
− 5	< 45	30	116	105	
− 4	*	50	100	89	
− 3	*	130	135	125	

*No value given in Xu and Jiang [35]
[1] According to Xu and Jiang [35], based on naked-eye Chinese sunspot records
[2] According to Schove [22]
[3] Based on 90 + 11 + 2 FIT
[4] Based on 50 + 11 + 2 FIT

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E. Cycles Beyond SCN 20

This report will now extrapolate the empirical function "forwards" in time, at least for cycles 21 and 22, the two cycles of more immediate concern. The 90 + 11 + 2 FIT predicts that cycle 21 has a 1-sigma confidence $\bar{R}_{MAX}$ value equal to 157.3 ± 18.8. This value is remarkably consistent with that predicted by Sargent [64] (= 154) and is very close to that which has now been observed (= 164.5). Table 5 lists estimates for $\bar{R}_{MAX}$ for cycle 21 based on all the approaches given earlier; it also gives residuals. It was seen that cycle 21 was about 0.38 s unit above the predicted value, in the average range of Figure 15. Table 6 lists the cycle-related parameters projected for cycle 21, based on the 90 + 11 + 2 FIT and on the actually observed $\bar{R}_{MAX}$ value. (Values for the cycle-related parameters are based on regression equations using $\bar{R}_{MAX}$ as the independent variable, given in Figures 18 through 20, included in the Appendix.) Also given in Table 6 are the occurrence dates for $\bar{R}_{MAX}$, $\bar{R}_{CHM}$ (ascent and descent side), and cycle end date, denoted here as $\bar{R}_{MIN}$(SCN + 1) occurrence which marks the beginning of the next cycle (SCN 22).

TABLE 5. $\bar{R}_{MAX}$ (COMP) VALUES FOR SCN 21

METHOD	$\bar{R}_{MAX}$(COMP) ± 1 SIGMA	RES = $\bar{R}_{MAX}$(OBS) – $\bar{R}_{MAX}$(COMP)
MEAN	116.2 ± 36.7	48.3
LR	130.1 ± 36.1	34.4
LR + 8	130.1 ± 33.4	34.4
LR + 11	142.8 ± 25.2	21.7
LR + 11 + 8	142.8 ± 24.9	21.7
LR + 11 + 2	158.6 ± 19.7	5.9
50	118.8 ± 37.0	45.7
50 + 8	118.8 ± 33.8	45.7
50 + 11	132.6 ± 24.5	31.9
50 + 11 + 8	132.6 ± 23.8	31.9
50 + 8 + 2	131.6 ± 31.2	32.9
50 + 11 + 2	148.0 ± 19.0	16.5
90	126.8 ± 36.1	37.7
90 + 8	126.8 ± 33.1	37.7
90 + 11	141.5 ± 24.6	23.0
90 + 8 + 2	140.0 ± 30.4	24.5
90 + 11 + 2	157.3 ± 18.8	7.2

Table 6 reveals that, based on the 90 + 11 + 2 FIT $\bar{R}_{MAX}$ projection (= 157.3), cycle 21 maximum would have been estimated to have occurred about February 1980 and $\bar{R}_{CHM}$ occurrence dates about October 1978 and January 1983; cycle end date would be expected to occur about March 1987. Based on actual dates of occurrence, it is noted that $\bar{R}_{MAX}$ occurrence would have been missed by only 2 months (December 1979), and $\bar{R}_{CHM}$ ascent-side and descent-side occurrences by 4 months and 2 months, respectively (June 1978 and March 1983). Cycle 21 is still in decline, so a comparison between computed and observed cycle end dates is not yet possible. Wilson [44] and Wilson et al. [45,100] have discussed the expected cycle duration for cycle 21. If cycle 21 is a short-period cycle (cycle duration less than 128 months),

TABLE 6. SELECTED PARAMETRIC VALUES FOR SCN 21

PARAMETER	MEAN VALUE[1]	STANDARD DEVIATION[1]	OBS VALUE	COMP. VALUE[2]	COMP. VALUE[3]
$\bar{R}_{MAX}$	116.2	36.7	164.5	157.3	
$\bar{R}_{MIN}$	5.2	2.7	12.2	6.1	6.2
$\bar{R}_{MEAN}$	53.4	16.2		70.7	73.7
S_{MEAN}	35.9	12.2		48.3	50.4
$\bar{R}_{CHM}$	60.7	18.8	88.4	81.7	85.4
ASC_{CHM}	26.3	5.8	24	27.6	27.8
D_{CHM}	56.1	9.2		50.9	50.0
ASC	48.2	6.2	42	44.3	43.6
DES	83.5	10.0		84.5	84.7
MIN—MIN PERIOD	131.6	10.0		128.8	128.3
$SLOPE_{ASC}$	2.398	0.958	3.626	3.426	3.606
$SLOPE_{DES}$	−1.337	0.441		−1.767	−1.851

			OBS.	COMP.[2]	COMP.[3]
$\bar{R}_{MAX}$ OCCURRENCE DATE:			DEC 1979	FEB 1980	
$\bar{R}_{CHM}$ OCCURRENCE DATES:		ASC—SIDE	JUN 1978	OCT 1978	OCT 1978
		DES—SIDE	MAR 1983	JAN 1983	DEC 1982
$\bar{R}_{MIN}$ (SCN + 1) OCCURRENCE DATE:				MAR 1987	FEB 1987

[1]BASED ON CYCLES 8 THROUGH 20

[2]BASED ON 90 + 11 + 2 FIT; $\bar{R}_{MAX}$(COMP) = 157.3

[3]BASED ON $\tilde{R}_{MAX}$(OBS) = 164.5

then cycle 21 should end before March 1987 (best-guess date: August 1986); otherwise, cycle 21 will be a long-period cycle (cycle duration greater than 133 months) ending no earlier than August 1987 (best-guess date: February 1988).

In Table 7, computed $\bar{R}_{MAX}$ values with 1-sigma confidence for cycle 22 are listed. Extrapolation of the 90 + 11 + 2 FIT yields an estimate of $\bar{R}_{MAX}$ for cycle 22 to be 107.4. (Sargent and Heckman [66] and Sargent [171] have discussed current efforts to predict cycle 22; also, a Solar Terrestrial Prediction Workshop with the expressed objective of discussing current efforts to predict cycle 22 occurred 18-22 June 1984 in Meudon, France.) Using this value, Table 8 projects values for the cycle-related parameters for cycle 22. The $\bar{R}_{MAX}$ occurrence date is projected to be

TABLE 7. $\bar{R}_{MAX}$ (COMP) VALUES FOR SCN 22

METHOD	$\bar{R}_{MAX}$(COMP) $\pm$ 1 SIGMA
LR + 11 + 2 FIT	111.9 $\pm$ 19.7
50 + 11 + 2 FIT	98.0 $\pm$ 19.0
90 + 11 + 2 FIT	107.4 $\pm$ 18.8
90 + 8 + 2 FIT	100.8 $\pm$ 24.5

TABLE 8. SELECTED PARAMETRIC VALUES FOR SCN 22

PARAMETER	MEAN VALUE[1]	STANDARD DEVIATION[1]	COMPUTED VALUE[2]
$\bar{R}_{MAX}$	116.2	36.7	107.4
$\bar{R}_{MIN}$	5.2	2.7	5.0
$\bar{R}_{MEAN}$	53.4	16.2	49.7
S_{MEAN}	35.9	12.2	33.3
$\bar{R}_{CHM}$	60.7	18.8	56.2
ASC_{CHM}	26.3	5.8	26.0
D_{CHM}	56.1	9.2	57.2
ASC	48.2	6.2	49.0
DES	83.5	10.0	83.2
MIN—MIN PERIOD	131.6	10.0	132.2
$SLOPE_{ASC}$	2.398	0.958	2.178
$SLOPE_{DES}$	−1.337	0.441	−1.245

	COMP.
$\bar{R}_{MIN}$ OCCURRENCE DATE:	MAR 1987
$\bar{R}_{MAX}$ OCCURRENCE DATE:	APR 1991
$\bar{R}_{CHM}$ OCCURRENCE DATES: ASC—SIDE	MAY 1989
DES—SIDE	FEB 1994
$\bar{R}_{MIN}$(SCN + 1) OCCURRENCE DATE:	MAR 1998

[1]BASED ON CYCLES 8 THROUGH 20

[2]BASED ON 90 + 11 + 2 FIT; $\bar{R}_{MAX}$(COMP) = 107.4

April 1991, $\overline{R}_{CHM}$ occurrence dates May 1989 and February 1994, and the cycle-end date March 1998, all dates based on the March 1987 projected end date for cycle 21. Other dates, based on bifurcation of sunspot cycles into short-period and long-period cycles, have been summarized elsewhere (Wilson et al. [45,100]). In contrast to cycle 21 which is the second highest documented $\overline{R}_{MAX}$ cycle, cycle 22 is projected to be a low-valued $\overline{R}_{MAX}$ cycle, very probably of long cycle duration. Future cycles are projected to be decreasing in $\overline{R}_{MAX}$ value through at least cycle 24 and then to be increasing in $\overline{R}_{MAX}$ value thereafter, at least through cycle 30 (Fig. 16). The next "Grand Maximum" like period is projected to be around SCN 40, corresponding to ca. A.D. 2180.

IV. CONCLUSIONS

This study has shown that values of $\overline{R}_{MAX}$ for modern sunspot cycles (i.e., cycles 8 through 21) can be satisfactorily fitted using a three-component sine curve representation, having an 11-cycle and a 2-cycle variation imbedded in an approximately 90-cycle periodicity (in terms of sunspot cycle number). The 90 + 11 + 2 FIT yields "minima," non-zero, low-value $\overline{R}_{MAX}$ periods, such as the Maunder and Spörer Minima, and a "maximum," high-value $\overline{R}_{MAX}$ period, such as the Grand Maximum, as reported by Eddy [3,4], although the values of $\overline{R}_{MAX}$ projected for the protracted sunspot minimum periods are considerably higher than has been estimated by other investigators [8,35]. Annual sunspot numbers, as reported by Xu and Jiang [35], for the Maunder Minimum are found to be very close to values "post-dicted" by the 90 + 11 + 2 FIT, especially if we consider the Maunder Minimum (and other such minima) to be a brief period when sunspot number is about 1-sigma below "normal" (i.e., below computed values). The 90 + 11 + 2 FIT, extrapolated forward in time, has been used to give estimates of $\overline{R}_{MAX}$ for cycles 21 and 22, and a number of useful linear relationships based on $\overline{R}_{MAX}$ [44] has been applied to project the behavior of these cycles. A comparison of observed and predicted values for the limited number of known cycle-related parameters for cycle 21 yields very good agreement. Cycle 22 is predicted to be a low-valued $\overline{R}_{MAX}$/long-period cycle, with $\overline{R}_{MAX}$ equal to about 107.

$SLOPE_{ASC} = -0.507 + 0.025 \, \overline{R}_{MAX}$

$r = 0.96, S_{yx} = 0.30$

$\overline{R}_{CHM} = 1.352 + 0.511 \, \overline{R}_{MAX}$

$r = 1.00, S_{yx} = 1.42$

$\overline{R}_{MEAN} = 4.488 + 0.421 \, \overline{R}_{MAX}$

$r = 0.95, S_{yx} = 5.27$

$\overline{R}_{MIN} = 2.624 + 0.022 \, \overline{R}_{MAX}$

$r = 0.29, S_{yx} = 2.85$
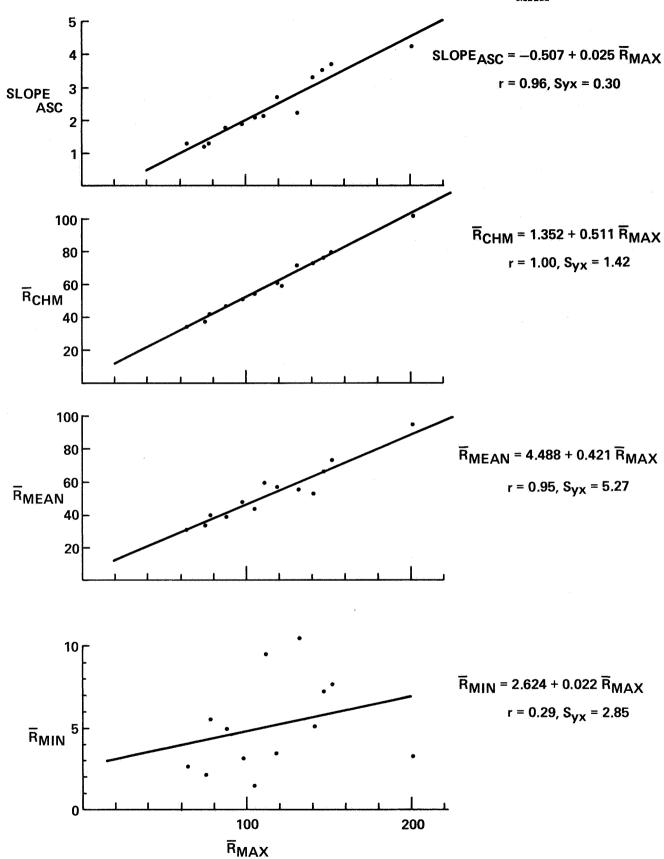
Figure 18. $SLOPE_{ASC}$, $\overline{R}_{CHM}$, $\overline{R}_{MEAN}$ and $\overline{R}_{MIN}$ versus $\overline{R}_{MAX}$.
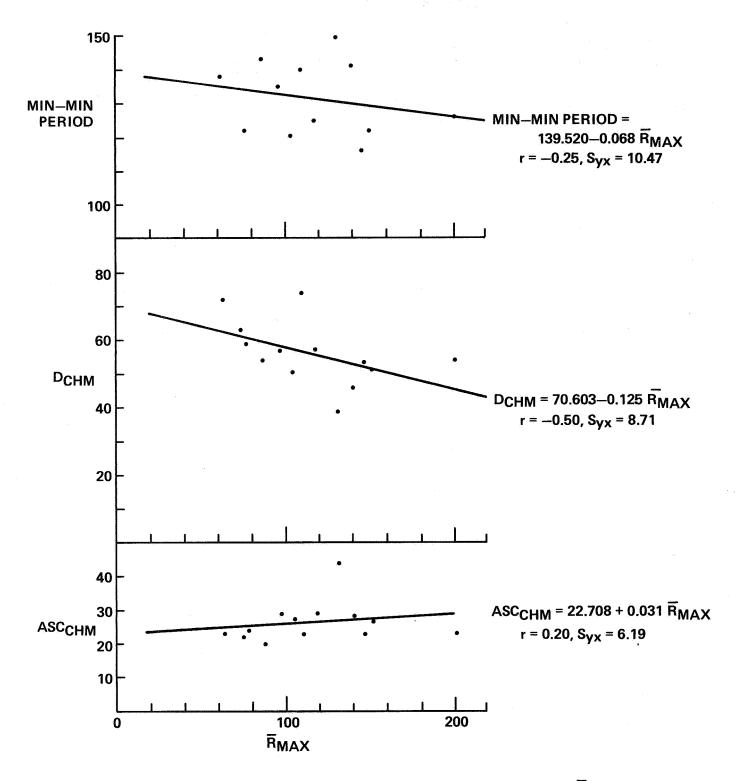
Figure 19. MIN-MIN PERIOD, D_{CHM} and ASC_{CHM} versus $\bar{R}_{MAX}$.

MIN-MIN PERIOD =
139.520−0.068 $\bar{R}_{MAX}$
r = −0.25, S_{yx} = 10.47

D_{CHM} = 70.603−0.125 $\bar{R}_{MAX}$
r = −0.50, S_{yx} = 8.71

ASC_{CHM} = 22.708 + 0.031 $\bar{R}_{MAX}$
r = 0.20, S_{yx} = 6.19

28
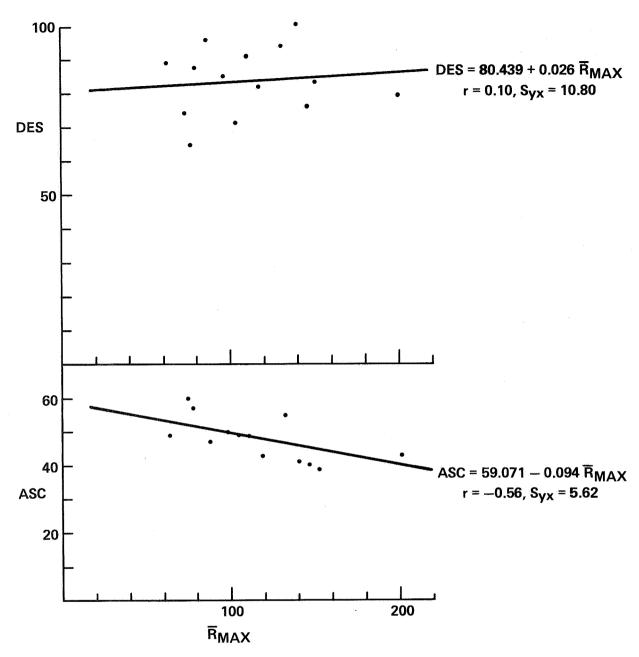
Figure 20. DES and ASC versus $\bar{R}_{MAX}$.

$$DES = 80.439 + 0.026\,\bar{R}_{MAX}$$
$$r = 0.10,\ S_{yx} = 10.80$$

$$ASC = 59.071 - 0.094\,\bar{R}_{MAX}$$
$$r = -0.56,\ S_{yx} = 5.62$$
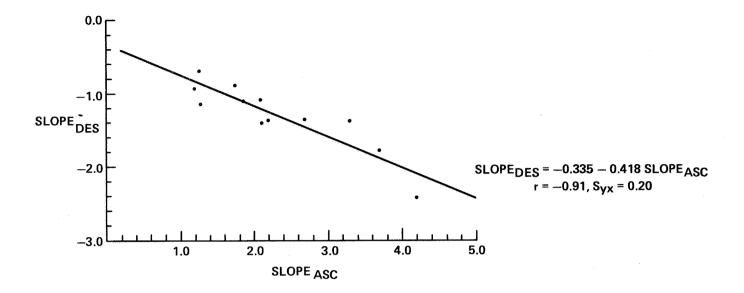
Figure 21. $SLOPE_{DES}$ versus $SLOPE_{ASC}$.

$$SLOPE_{DES} = -0.335 - 0.418 \, SLOPE_{ASC}$$
$$r = -0.91, \, S_{yx} = 0.20$$
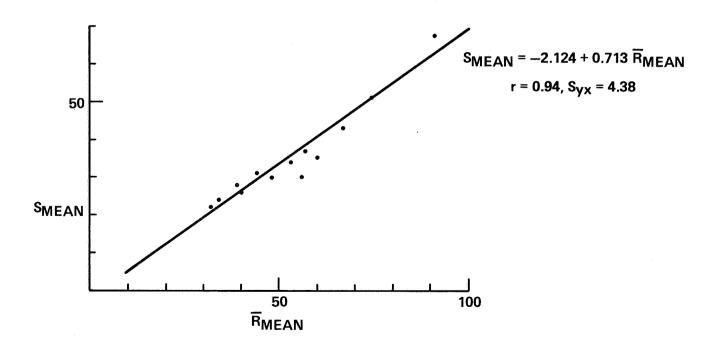
Figure 22. s_{MEAN} versus $\overline{R}_{MEAN}$.

$$S_{MEAN} = -2.124 + 0.713 \, \overline{R}_{MEAN}$$
$$r = 0.94, \, S_{yx} = 4.38$$

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APPROVAL

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By Robert M. Wilson

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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